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Trieste october 30, 1995

Dear Michael,

here is, with some delay, our final version of what we would like to say about the EPR-problem and the issue of ER-Loc. Obviously we have written the paper from our point of view and we are open to all changes you consider necessary, provided the two conditions I have stated in my previous letter are satisfied, as you agreed.

Some remarks:

We have not made any particular effort in writing the paper in good english since we know that you will change it and improve it a lot.

We have not suggested, in the introduction, any link between the problem we are discussing and Bohm's views. We know that you have appropriate ideas about the way to do this.

We have inserted practically only references to our previous papers simply to point out which quotations we would like to appear for what concerns us. Obviously we leave to you the addition of the relevant quotations of your work, as well as other quotation you consider important.

Obviously we are very interested in hearing from you what do you think of our argument and we are ready to include changes in the presentation if you consider them necessary. We hope that you will agree about the "logic" of our arguments and our conclusions.

In case there is a fundamental agreement between us, we are anxious to receive the proposed final version to be submitted for publication.

I hope that with your efforts we will have a good common paper included in the Bohm volume.

With my best regards

GianCarlo Ghirardi

GianCarlo Ghirardi

G.C. Ghirardi, R. Grassi, P. La Reviere and M. Redhead

## 1. Introduction.

In a recent paper (Ghirardi and Grassi 1994, 397) the EPR-argument has been reconsidered within a relativistic framework by taking appropriately into account the most striking (in our opinion) implication of the quantum character of natural processes, i.e., nonlocality. The main aim of the above paper was to investigate whether one can claim that the assumptions of validity and completeness of quantum mechanics imply a "spooky action-at-a-distance". It has been shown that this is not the case.

Here we will revisit the matter with two purposes. First of all we will make more precise and we will disucuss in greater details two logically independent locality assumptions which, even though correctly formulated, have not been sufficiently emphasized in the above paper. As a consequence the present argument will be more direct and concise. Secondly we will show that, when one appropriately limits his considerations to *objectively possessed or accessible** properties, the conclusion of (Ghirardi and Grassi 1994, 397) can even be reinforced, in the sense that not only one cannot state that there is instantaneous creation of elements of physical reality at-a-distance, but actually one can state that no effect of this type and, more generally, no parameter dependence effect takes place.

### 1.1 General assumptions and the locality issue.

Let us begin by fixing our assumptions and notations:

**Q.M.**: is the assumption that the predictions of Quantum Mechanics about the outcomes of measurement processes on the system we will deal with are correct.

**Compl.**: is the completeness constraint which, according to EPR, any theory must satisfy. It demands that the theory must contain the formal counterpart of any element of physical reality (e.p.r.) that is recognized as existent (see below for a detailed discussion). It goes without saying that, in the case of standard quantum mechanics the statevector itself is assumed to represent the most detailed possible description of the state of an individual physical system.

The nonlocal aspects of the quantum formalism will play a fundamental role in our analysis, so it is useful to recall their main features. For simplicity we will confine our considerations to an EPR-Bohm like situation and, in the spirit of Bell's analysis, we will assume that the complete specification of the state of an individual physical system is given by appropriate parameters $\lambda$ (which may include the quantum mechanical statevector or even reduce to it alone). We will denote by $p_\lambda^{LR}(x,y;\mathbf{n},\mathbf{m})$ the joint probability of getting, for the complete specification of the state of the system given by $\lambda$, the outcome $x$ ($x=\pm 1$) in a measurement of the spin component along $\mathbf{n}$ at the left (L) wing of the

*For a precise definition of this expression see the paper by Ghirardi and Grassi in this volume and Section 6 of the present paper

apparatus and the outcome y ($y=\pm 1$) in a measurement of the spin component along **m** at the right (R) wing. The marginal probability distributions for the outcome x along **n** at L when a measurement along **m** takes place at R, and similarly the one for the outcome y along **m** at R when a measurement along **n** takes place at L, will be denoted by $p_\lambda^L(x;\mathbf{n},\mathbf{m})$ and $p_\lambda^R(y;\mathbf{n},\mathbf{m})$, respectively. Such quantities are obviously given by:

$$p_\lambda^L(x;\mathbf{n},\mathbf{m}) = \sum_{y=-1,+1} p_\lambda^{LR}(x,y;\mathbf{n},\mathbf{m}); \quad p_\lambda^R(y;\mathbf{n},\mathbf{m}) = \sum_{x=-1,+1} p_\lambda^{LR}(x,y;\mathbf{n},\mathbf{m}) \qquad (1.1)$$

Finally, we will denote by $p_\lambda^L(x;\mathbf{n},*)$ and $p_\lambda^R(y;*,\mathbf{m})$ the probability of the considered outcome at the considered wing when no measurement is performed at the other wing.

Bell's locality assumption (**B-Loc**) is then expressed as:

$$\textbf{B-Loc}:\ p_\lambda^{LR}(x,y;\mathbf{n},\mathbf{m}) = p_\lambda^L(x;\mathbf{n},*)\cdot p_\lambda^R(y;*,\mathbf{m}). \qquad (1.2)$$

Some remarks are appropriate:

**i.** Assumption (1.2) has been proved (Suppes and Zanotti 1976, 44; van Fraassen 1982, 25; Jarrett 1984, 569) to be equivalent to the conjunction of two logically independent conditions:

a. Parameter Independence (**P.I.**):

$$\textbf{P.I.}:\ p_\lambda^L(x;\mathbf{n},\mathbf{m}) = p_\lambda^L(x;\mathbf{n},*); \quad p_\lambda^R(y;\mathbf{n},\mathbf{m}) = p_\lambda^R(y;*,\mathbf{m}) \qquad (1.3a)$$

b. Outcome Independence (**O.I.**):

$$\textbf{O.I.}:\ p_\lambda^{LR}(x,y;\mathbf{n},\mathbf{m}) = p_\lambda^L(x;\mathbf{n},\mathbf{m})\cdot p_\lambda^R(y;\mathbf{n},\mathbf{m}) \qquad (1.3b)$$

expressing that the probability of an outcome at L (R), is independent from the setting chosen at R (L), and that the probability of an outcome at one wing does not depend on the outcome which is obtained at the other wing, respectively. As is well known standard quantum mechanics meets the **P.I.** requirement while violating **O.I.**

**ii.** Since, as a consequence of Bell's analysis, the quantum predictions imply a violation of **B-Loc**, there follows that

$$\textbf{Q.M.} \supset \neg\textbf{P.I.} \vee \neg\textbf{O.I.}. \qquad (1.4)$$

**iii.** Given that the violation of **B-Loc** does not imply (Eberhard 1978, 329; Ghirardi Rimini and Weber 1980, 293) the possibility of faster-than-light signalling (thus avoiding an unacceptable crash between quantum mechanics and relativity) it has to be stressed that, from a conceptual point of view, the violation of **P.I.** is more disturbing than the violation of **O.I.**. In fact, as appropriately pointed out by Jarrett, a violation of **P.I.** implies that, would one have access to the parameter $\lambda$, events in a given

space-time region might be "influenced" by controllable actions which could be performed at the experimenter's whim in space-like separated regions.

## 1.2 Property attribution and elements of physical reality.

The second essential aspect we will be concerned with is that of property attribution to individual physical systems, which represents the crucial point of the EPR-argument. In fact, the nonlocality issue acquires new interesting connotations if one puts forward the desideratum that one should be allowed, at least under appropriate circumstances, to speak of properties or elements of physical reality (e.p.r), objectively possessed by individual microscopic physical systems. According to the appropriate EPR suggestion this, in a nonrelativistic context, should be related to the possibility of "predicting" the outcome of a prospective measurement.

For clarity sake we briefly reconsider this problem within nonrelativistic standard quantum mechanics with the postulate of wave packet reduction. We will limit ourselves to few general remarks.

**i.** One can adopt as a sufficient criterion for property attribution, i.e., for being allowed to claim that there exists an (e.p.r.) objectively possessed by an individual physical system Einstein's celebrated criterion: *If, without in any way disturbing a system we can predict with certainty ( i.e., with a probability equal to unity) the value of a physical quantity, then there exist an element of physical reality corresponding to this physical quantity.*

**ii.** This new way of looking at our problem requires reconsidering the previous analysis about the role played by the assumptions of **P.I.** and of **O.I.**. In fact, would the EPR inference that the mere knowledge of the outcome of a measurement in a certain space time region makes legitimate the attribution of (e.p.r.) in a space-like separated region hold unchanged even within a relativistic and nonlocal context, one could legitimately claim that the factual situation would allow one to decide, at free will, whether to create instantaneously a property of a system far away, by switching on an apparatus.

As we did before, let us use a notational shortcut for the locality request that one cannot influence the (e.p.r) at-a-distance:

**ER-Loc**: is the assumption that elements of physical reality referring to an individual physical system cannot be changed by actions taking place in space-like separated regions.

It goes without saying that in the nonrelativistic case the EPR-argument can be read as establishing that:

$$Q.M. \wedge Compl. \supset \neg(ER - Loc), \tag{1.5}$$

which makes understandable why Einstein was lead to speak of “spooky action at-a-distance”.

## 1.3 The conclusions of Ghirardi and Grassi.

We are now ready to make precise the point which has been raised and discussed in (Ghirardi and Grassi 1994, 397): is Einstein's claim that **Q.M.** and **Compl.** imply a violation of **ER-Loc** legitimate also within a relativistic context? If this would be the case, the common statement that due to the fact that the theory violates **B-Loc** by violating only **O.I.** quantum mechanics can peacefully cohexist with relativity would be quite inappropriate. In fact, a violation of **ER-Loc** must be considered as logically equivalent to a violation of **P.I.**. The conclusion of the above quoted paper, which we will rederive here in a simpler and more clean way, is that **it is not possible** to draw from the assumptions of **Q.M.** and **Compl.** the inference that **ER-Loc** is violated*.

Before concluding this Section we mention that, as already anticipated, in the last part of the paper we will show that **ER-Loc** is implied by Q.M.$\wedge$Compl if one limits his considerations to *objectively possessed or accessible* properties.

## 2. Attribution of elements of physical reality to individual physical systems in the relativistic case.

Within a relativistic context, since there is no absolute time ordering, the very expression *predict* appearing under **i.** in the previous section is meaningless. As it has been suggested by various authors (see, e.g., (d'Espagnat 1984, 110)), and how it has been discussed in great detail in (Ghirardi and Grassi 1994, 397) to deal with this matter for the relativistic case it is appropriate to relate the possibility of attributing (e.p.r.) to an individual physical system to the validity of an appropriate counterfactual statement. To this purpose, following (Ghirardi and Grassi 1994, 397), let us introduce the following notation:

$\mathcal{M}_S^A(t)$: A measurement of the observable $\mathcal{A}$ of the physical system S is performed at time t.

$O_S^{A=a}(t)$: the outcome of $\mathcal{M}_S^A(t)$ is $\mathcal{A}$=a.

$\mathcal{P}_S^{A=a}(t)$: The system S possesses, at time t, the (e.p.r.) $\mathcal{A}$=a.

We can then formulate the criterion for property attribution in a way which is appropriate for our purposes.

*Criterion for property attribution*: We relate the possibility of attributing an (e.p.r.) to an individual physical system to the validity of a counterfactual statement:

**P.A.1.**: $[\mathcal{M}_S^A(t) \;\Box\!\!\rightarrow O_S^{A=a}(t)] \supset \mathcal{P}_S^{A=a}(t)$.

More generally, we will state that the system possesses an (e.p.r.) referring to the observable $\mathcal{A}$ and we will write $\mathcal{P}_S^A(t)$ according to:

*To avoid being misunderstood we warn immediately the reader that we are not claming that the premisses allow one to prove that ER-Loc is not violated.

**P.A.2.:** $\mathcal{P}_S^{A}(t) \equiv_{df} \bigvee_{a_k \in Sp(\hat{A})} \mathcal{P}_S^{A=a_k}(t)$,

where we have denoted by $Sp\{\hat{A}\}$ the spectrum (which for simplicity is assumed to be purely discrete) of the self-adjoint operator $\hat{A}$ associated to $\mathcal{A}$.

## 3. The Principle of Local Counterfactual Definiteness: some comments.

As is well known the legitimacy of using, within a genuinely stochastic context, the Principle of Local Counterfactual Definitiness (PLCD) claiming (Redhead 1987, 92) that "the result of an experiment which *could* be performed on a microscopic system has a definite value which does not depend on the setting of a remote piece of apparatus", is rather controversial. Different authors have taken quite different positions about this problem (Redhead 1982, 92 ; Lewis 1973). We do not intend to enter in this delicate matter for two important reasons which we are going to expouse.

i. In the nonrelativistic case in which there is an absolute time ordering, it is natural and generally accepted to use the criterion of inevitability at time t to characterize the accessibility spheres. This implies that one does not need to resort to PLCD to state that if the system is in the singlet spin state and a measurement of $\boldsymbol{\sigma}^R \cdot \mathbf{n}$ has been performed at R at time t and has given, e.g., the outcome +1 then the counterfactual statement referring to a subsequent time t':

$$[\mathcal{M}^L(t') \Box\!\!\rightarrow O^{\sigma^L \cdot n = -1}(t')] \tag{3.1}$$

is true.

ii. The second reason for not worrying here about the use of PLCD in a relativistic context stems directly from the fact that the logical structure of the argument that we will develop can be summarized in the following terms: for the problem we are discussing, even if (PLCD) is adopted, the combined role of relativity and nonlocality will not allow one to draw the conclusion* that the measurement at R implies the istantaneous emergence of an (e.p.r.) at L.

## 4. Deepening the argument: the assumption of OM-Loc

To better understand the line of thought we will follow it turns out to be useful to make a specific locality assumption about the macroscopic outcomes in individual measurement processes which is, from a conceptual point of view, slightly different from the one of **B-Loc**. The difference arises from the fact that **B-Loc**, as previously formulated, refers fundamentally to the probabilities of various outcomes, while we need an

*It goes without saying that, if one refuses PLCD, then one simply states that nothing can be said and reaches (rather trivially and less significantly) the same conclusion.

assumption directly related to the occurrence of a specific outcome in a specific process.

The distinction is rather subtle and could be considered as superfluous, since, as we will show, the request is practically equivalent to **B-Loc**, once one accepts that nature is not particularly malicious in cheating us about physical processes. Let us make our assumption explicit:

**OM-Loc**: is the assumption that the outcome obtained in a measurement cannot depend by measurements performed in space-like separated regions.

Some brief comments.

**i.** In (Ghirardi and Grassi 1994, 397) the assumptions of **ER-Loc** and of **OM-Loc** have not been kept distinct in the formulation of the locality requirement appropriate for the relativistic analysis. The two assumptions have been listed together as a way of articulating the request they have called **L-Loc** (for Lorentz locality), without stressing that this assumption is the logical conjunction of the two logically independent assumptions we are dealing with, i.e. **ER-Loc** and **OM-Loc**. Even though in the development of their argument the authors have been cautious and have used one or the other assumption at the appropriate places, having not made clear the difference between them has made their argument less simple than the one we will develop here.

**ii.** It is easy to convince oneself that **B-Loc** is equivalent to **OM-Loc** banning a conspiracy of nature to cheat us. In fact, $\mathrm{B-Loc} \supset \mathrm{OM-Loc}$ unless the individual events at the two wings influence each other in a very peculiar way which however respects the factorization of the joint probabilities. As a simple example one can consider the state $|\sigma_x^R = +1 > |\sigma_x^L = +1 >$ and one can suppose that in a measurement of $\sigma_z$ all outcomes are reversed when two measurement are performed with respect to the case of only one measurement.

Similarly $\mathrm{OM-Loc} \supset \mathrm{B-Loc}$, unless occasional coincidences of outcomes lead to an agreement, in all actual cases, with the correlations predicted by quantum mechanics; an occurrence that, even if conceivable in principle, can safely be ignored.

## 5. The main argument and conclusion of the paper

With the premisses of the preceding sections, it is now straightforward to derive our conclusions. For notational simplicity we will not specify the direction along with the spin measurement is performed, giving for granted that we will always consider the direction to be the same at the two wings of the apparatus. We warn the reader that in the first part of our argument we will assume tentatively that **OM-Loc** licenses for PLCD. However, as already remarked, and as it will appear clear in what follows, the conclusion we will draw does not depend on this assumption.

Let us then go through the argument using PLCD:

1. $\mathcal{M}_R^{\sigma}(t_R)$: a measurement of the spin component is performed at R at time $t_R$.

2. $\mathcal{M}_R^{\sigma}(t_R) \supset \mathbf{O}_R^{\sigma=+1}(t_R) \vee \mathbf{O}_R^{\sigma=-1}(t_R)$

3a1. $[\mathrm{Q.M.} \wedge \mathbf{O}_R^{\sigma=+1}(t_R) \wedge \mathrm{OM-Loc}] \supset [\mathcal{M}_L^{\sigma}(t_L) \Box\!\!\rightarrow \mathbf{O}_L^{\sigma=-1}(t_L)]$

3a2. $[\mathcal{M}_L^{\sigma}(t_L) \Box\!\!\rightarrow \mathbf{O}_L^{\sigma=-1}(t_L)] \supset \mathcal{P}_L^{\sigma=-1}(t_L)$

3b1. $[\mathrm{Q.M.} \wedge \mathbf{O}_R^{\sigma=-1}(t_R) \wedge \mathrm{OM-Loc}] \supset [\mathcal{M}_L^{\sigma}(t_L) \Box\!\!\rightarrow \mathbf{O}_L^{\sigma=+1}(t_L)]$

3b2. $[\mathcal{M}_L^{\sigma}(t_L) \Box\!\!\rightarrow \mathbf{O}_L^{\sigma=+1}(t_L)] \supset \mathcal{P}_L^{\sigma=+1}(t_L)$

4. $[\mathrm{Q.M.} \wedge \mathrm{OM-Loc}] \supset \mathcal{P}_L^{\sigma}(t_L)$

5. At time $t_L$, and for the particle at L, $\exists$[(e.p.r.)- associated to $\sigma$]

6. $\mathrm{ER-Loc} \supset \exists$[(e.p.r.)- associated to $\sigma$] even if $\neg\mathcal{M}_R^{\sigma}(t_R)$

7. If no measurement is performed at R, the statevector is the singlet which does not contain any formal element referring to such an (e.p.r.), i.e. $\neg$Compl..

Summarizing we have proved that, if one accepts PLCD, then:

$$\mathrm{Q.M.} \wedge (\mathrm{OM-Loc}) \wedge (\mathrm{ER-Loc}) \supset \neg\mathrm{Compl.} \tag{5.1}$$

or, equivalently:

$$\mathrm{Q.M.} \wedge \mathrm{Compl.} \supset \neg(\mathrm{OM-Loc}) \vee \neg(\mathrm{ER-Loc}). \tag{5.2}$$

Here comes the conclusive step. Eq.(5.2) shows that, if one assumes $\mathrm{Q.M.} \wedge \mathrm{Compl.}$, three alternatives are still possible:

a). OM-Loc is false and ER-Loc holds true,

b). OM-Loc is true and ER-Loc is false,

c). Both OM-Loc and Er-Loc are false.

It is extremely important to stress that alternative a) cannot be excluded. In fact let us assume that OM-Loc is violated. As a consequence, the very first steps of the above analysis, i.e. the assumption that for all accessible worlds the specific outcome at $(R, t_R)$ is the same as the one, e.g., $\mathbf{O}_R^{\sigma=+1}(t_R)$ which has occurred in the actual world, cannot be mantained. In fact, accepting nonlocality amounts exactly to admitting that the outcome at $(R, t_R)$ might depend on the occurrence of $\mathcal{M}_L^{\sigma}(t_L)$. As a consequence, the fact that in the actual world, in which only the measurement at R takes place, $\mathbf{O}_R^{\sigma=+1}(t_R)$ holds true does not justify, by itself, either the counterfactual statement:

$$[\mathcal{M}_L^{\sigma}(t_L)\Box\!\!\rightarrow O_L^{\sigma=-1}(t_L)] \supset \mathcal{P}_L^{\sigma=-1}(t_L) \tag{5.3a}$$

or its opposite, i.e., the statement:

$$[\mathcal{M}_L^{\sigma}(t_L)\Box\!\!\rightarrow O_L^{\sigma=-1}(t_L)] \supset \mathcal{P}_L^{\sigma=+1}(t_L). \tag{5.3b}$$

This proves that one cannot exclude alternative a). Stated differently, we have proved that, even if one would accept PLCD, within a relativistic context, there is no way to use an EPR-type argument to prove that $Q.M.\wedge Compl.$ imply a violation of ER-Loc. Recalling that standard quantum mechanics does not violate P.I. there follows that one cannot exclude the simoultaneous validity of $Q.M.\wedge Compl \wedge P.I.\wedge Er-Loc$.

## 6. A more general argument and an illuminating example.

In this section we will reconsider the whole matter by making reference to the requests that the consideration of dynamical reduction models has naturally lead to put forward for considering a property as possessed by an individual physical system..

### 6.1 Objectively possessed or accessible properties and its consequences.

In (Ghirardi, Grassi and Benatti 1995, 25) a critical analysis of the criteria for property attribution has been performed. The appropriateness of revisiting this matter is suggested by the formal and conceptual aspects of recently proposed models of dynamical reduction aiming to overcome the problems that standard quantum mechanics meets in dealing with macroscopic phenomena. The essential points of the analysis have been summarized in the contribution by Ghirardi and Grassi to the present volume. The idea underlying this attempt to "close the circle within a genuinely quantum context" is quite simple. If one wants to transform the "fuzzy" quantum picture of reality into one which is "compatible with our experience about macroscopic processes", one must look for a modification of the standard formalism which, in particular, allows one to make precise the statement that a property or an (e.p.r.) is objectively possessed (accessible) by an individual physical system*. Let us make clear the precise meaning we attach to the above expressions.

*Definition*: we claim that a property corresponding to a value (a range of values) of a certain variable in a given theory is *objectively possessed or accessible* when, according to the predictions of that theory, there are unambiguous experimental procedures (or physical processes) yielding reliable information about the variable that would, if performed (or taking place), give an *outcome* corresponding to the claimed value. Thus, the crucial feature characterizing *accessibility* (as far as statements about individual physical sistems and/or processes are concerned) is the *matching* of the *claims and the outcomes* of the physical processes testing the claims.

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$$[\mathcal{M}_L^{\sigma}(t_L)\Box\!\!\rightarrow O_L^{\sigma=-1}(t_L)]\supset \mathcal{P}_L^{\sigma=-1}(t_L) \tag{5.3a}$$

or its opposite, i.e., the statement:

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This proves that one cannot exclude alternative a). Stated differently, we have proved that, even if one would accept PLCD, within a relativistic context, there is no way to use an EPR-type argument to prove that $Q.M.\wedge Compl.$ imply a violation of ER-Loc. Recalling that standard quantum mechanics does not violate P.I. there follows that one cannot exclude the simoultaneous validity of $Q.M.\wedge Compl \wedge P.I.\wedge Er-Loc$.

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This point is of particular relevance for our discussion. In fact, by taking the above precise position about properties one is relating the possibility of claiming that they are possessed to the validity of a counterfactual statement concerning the physical system under consideration. We warn the reader that we are no more relating the existence of (e.p.r.) to *our knowledge* about the system. We might be lacking the knowledge allowing us to claim that a specific property is objectively possessed (as it could happen, e.g., when we know that a spin measurement has taken place and a definite (unknown to us) outcome has been obtained) but nevertheless it could be true that an appropriate counterfactual statement about an observable referring to the individual system holds.

If one takes this attitude, then one immediately sees that the validity of a counterfactual statement about a physical system besides being, as in the previous argument, a sufficient condition for the existence of (e.p.r.), becomes also a necessary one.

One can then argue as follows: $Q.M.\wedge Compl.\supset \neg B-Loc$ , a fact that, banning untenable assumptions about nature, is equivalent to $Q.M.\wedge Compl.\supset \neg OM-Loc$. Let us assume now that $Compl.\wedge\neg(OM-Loc)$ hold, and suppose that, in a specific instance, a measurement has been performed at R and the outcome +1 (-1) has been obtained. If we are interested in counterfactual statements having this assumption as a premise, we must include in the accessibility sphere of the actual world, both worlds in which the same result and worlds in which the opposite result has been obtained. In fact, would this not be the case, this would mean that the wave function does not represent the maximal information one can have about an individual physical system* or that $(OM-Loc)$ holds. But, this being the case, it follows form the theory of counterfactual implication that no one of the two mutuallly incompatible and exhaustive statements $[\mathcal{M}_L^\sigma(t_L)\Box\!\!\rightarrow 0_L^{\sigma=-1}(t_L)]$ or $[\mathcal{M}_L^\sigma(t_L)\Box\!\!\rightarrow 0_L^{\sigma=+1}(t_L)]$ can be true. In turn, since the validity of the counterfactual statement has been shown to be also a necessary condition for claiming that the system objectively possesses an accessible (e.p.r.), one can state that no (e.p,r,) exists, and therefore that ER-Loc holds. The conclusion should now be clear: if one takes the position that only the accessible properties of individual physical systems deserve to be considered, then the above argument proves that the assumptions of $Q.M.\wedge Compl.\wedge P.I.\wedge Er-Loc$ hold. This concludes our analysis. The remaining part of the paper is devoted to show how it has actually been the critical reconsideration of the problems which arose in connection with the dynamical reduction program that has led two of us to derive the conclusions presented in (Ghirardi and Grassi 1994, 397) and extended in this paper.

### 6.2 The nonrelativistic models of dynamical reduction.

We will start once more by considering the nonrelativistic case (Ghirardi, Rimini and Weber 1986, 470; Ghirardi and Rimini 1990, 167; P. Pearle 1989, 2277; Ghirardi, Pearle and Rimini 1990, 78) but contemplating the possibility of a dynamical modification of the theory

*Stated differently we would be denying the Compl. assumption.

leading to spontaneous localizations. The nonlinear and stochastic features which are introduced in such an approach have as their basic feature that of striving to drive the statevector of any individual physical system within one of the eigenmanifolds of the so called preferred basis. However the process requires in principle an infinite time*. As a consequence, as discussed in all details in the quoted paper, within such a context one is compelled to resort to a slightly modified criterion for property attribution. One attributes to an indiviual physical system the (e.p.r.) corresponding to the value a of the observable $\mathcal{A}$, even when the statevector $|\Psi,t>$ describing the system at the indicated time is not exactly an eigenvector of the self-adjoint operator $\hat{A}$ corresponding to $\mathcal{A}$, but when the norm $\|P_a|\Psi,t>\|$ of the projection of the statevector on the eigenmanifold associated to a is extremely close to one. It is useful to stress that even within the standard theory one is led to the same conclusion, i.e. that the property $\mathcal{A}$=a must be attributed also when the above norm is not exactly equal but extremely close to one, i.e.:

$$[\|P_a|\Psi,t>\| \cong 1] \equiv \mathcal{P}_S^{A=a}(t). \qquad (6.1)$$

This is due to the fact that the outcome of a measurement is unavoidably related to the position of some macroscopic pointer, and no wave function can have a strictly compact support in configuration space. This remark allows one to make precise the vague statement "extremely close": see (Ghirardi, Grassi and Pearle 1990, 1271;Ghirardi and Pearle 1991, 35).

### 6.3 The relativistic case.

Even though there is not a perfectly satisfactory relativistic dynamical reduction model since all attempts in this direction have met some difficulties related to the appearence of divergences, it has however been possible to outline how such a theory should look, to identify the way in which the evolution should be described, to discuss the problem of stochastic relativistic invariance and to identify some general features that any such model should unavoidably exhibit (Ghirardi, Grassi and Pearle 1990, 1271; Ghirardi and Pearle 1991,35).

The most relevant point for our concerns here derives from the fact that, within the considered framework, the mean value of a local observable $\mathcal{A}$ having compact support on a space-like hypersurface may depend (at the individual level, not at the ensemble level) from the specific surface, between all those containing the support of $\mathcal{A}$, that one chooses to evaluate it. If, within such a context, one wants to maintain that the statevector accounts for the properties possessed by an individual physical system, one is lead (Ghirardi, Grassi and Pearle 1990, 20), to modify criterion for the attribution of (e.p.r.). It has been shown that the appropriate criterion is the following: an individual physical systems can be claimed to possess the value a for a local observable $\mathcal{A}$ when $[\|P_a|\Psi(\sigma)>\| \cong 1]$ holds for all space-like hypersufaces $\sigma$ which contain the support of $\mathcal{A}$. The model and the criteria under discussion have been analyzed in great detail in a series of papers, among the others in

*For a detailed discussion of this point see (Ghirardi, Grassi and Pearle 1990, 1271)

(Ghirardi, Grassi, Butterfield and Fleming 1993, 341; Butterfield, Fleming, Ghirardi and Grassi 1993, 2287) and it has been proved that the resulting theoretical scheme violates P.I. and ER-Loc only in absolutely exceptional cases corresponding to the occurrence of an innumerable sequence of stochastic events each of which has an almost vanishing probability. If one ignores such cases, the proposed criterion for the attribution of (e.p.r.) is equivalent (with the same degree of accuracy) to the counterfactual statement $\mathcal{M}_S^A \Box\!\!\rightarrow O_S^{A=a}$.

Before concluding we want to call attention on another possibility (Ghirardi, Grassi and Pearle 1990, 20) for a criterion allowing one to attribute, within the consideered thoeretical framework, an (e.p.r.) corresponding to a local observable $\mathcal{A}$ to an individual physical system. The criterion requires that $\| P_u | \Psi(\Sigma) > \|$ be extremely close to one, where $\Sigma$ is the space-like hypersurface consisting of the compact support of $\mathcal{A}$, the past light cone originating from it, and the part of the hypersurface on which the initial conditions are assigned lying otside the considered light cone. One can easily check that this criterion for the attribution of (e.p.r.) to an individual physical system is in any case equivalent to the one based on the validity of a counterfactual statement and that the resulting theory does not exhibit, in such a case, any violation of P.I. and any violation of ER-Loc. This discusion should have made more clear the argument of section 6.1.